THE CORRELATION OF HISTORICAL ECONOMIC VARIABLES AND THE MISUSE OF COEFFICIENTS IN THIS CONNECTION.

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He who reads statistical literature nowadays finds it literally teeming with studies in correlation. Apparently, the time has arrived when a statistician's ability is largely judged by his output of correlation coefficients. Unfortunately, however, many of the coefficients seem to be more valuable as mementoes of industry than for any new truth which they add to human knowledge. In fact, it may be said without exaggeration that only too often the statistician has been so entranced by the mathematical possibilities of the problem that he has lost sight of the real meaning of the operations involved and hence the conclusions presented have been utterly fallacious and entirely contrary to the facts. Is it not time to call a halt and ascertain just what ends may and may not be served by this newly popular mathematical aid?

It is apparently in the correlation of historical variables that statisticians have wandered astray most frequently. Economists have borrowed a new machine from mathematicians and have proceeded to use it without understanding its workings. As a result, strange and wonderful laws of economics have been deduced; figures have been made to lie most glibly, and all because the limitations of the device have not been recognized. Perhaps a brief explanation of correlation and its meaning may prove helpful to others who, like the present writer, may have searched long in the literature on the subject and found little enlightenment.

The first question is "What does the term correlation mean?" Evidently, it implies some relationship between things, and this particular relationship is always one of cause and effect. If a change in A causes a change in B, there is said to be correlation between them. Likewise if a variation in A causes both B and C to vary, there is said to be correlation between B and C. But, if B and C should, for entirely unconnected reasons, happen to vary in the same manner, there would be

no correlation between B and C, since the relationship is not a causal one.

One of the laws of physics is that a given force always produces identically the same effect whenever it is applied. Does this same rule hold good for economic or social phenomena? There is every reason to believe that it is exactly as valid here as in the other parts of the physical world. If this is not true, it is useless to hope for any material advancement of economic science. A law which only works part of the time is no law at all. Economics deals with physical beings and forces and it would be strange indeed if it should prove an exception to the general rules of the natural world. The chemist knows that 16 grams of oxygen and 2 grams of hydrogen will make exactly 18 grams of water. The physicist is certain that a force of one dyne will always give an acceleration of exactly one centimeter per second to one gram. They allow for no exceptions to the rule. The economist, on the other hand, has usually been satisfied to say that a lessening of the wheat supply by one tenth will increase the price, to an indefinite extent, perhaps 20 per cent., or perhaps 40 per cent. The natural scientist figures accurately; the economist ordinarily calculates but vaguely. Why is this true? The fact seems to be that the "natural" sciences are more highly developed and better understood than the science of economics. They are older and mathematics earlier lent them its aid. Economists are now really just beginning to utilize mathematics and their attempts in this line are still marked by the crudeness of the amateur.

One of the chief reasons why economists have put so little faith in the mathematical accuracy of the laws which they have formulated is that many of them have not fully comprehended the principle of the resultant of several forces acting simultaneously. They have failed utterly to appreciate or to utilize the concepts which are to the physicist the most elementary bases of the study of mechanics. They have attempted to deal with whole groups of variable forces at once instead of separating them carefully and studying the effects of each individual force by itself.

Isaac Newton long ago pointed out the fact that when several physical forces act on a body at the same time each

produces its own effect regardless of the others and that any motion arising is the resultant of all the independent forces compounded. Thus gravity, the bore of the gun, and the explosive force of the powder all act together in determining the movements of the projectile. The physicist has long ago separated these component forces and he can point out accurately the part played by each. Economists, on the other hand, are only beginning to realize that they must attack a problem in a manner identical with that used by the physicists.

To illustrate: If a steamboat which, in still water, would travel ten miles an hour proceeds upstream in a river flowing four miles per hour, the net result is but six miles an hour of actual progress. No physicist would think of attempting to measure the propulsive force of the engines by the distance traveled without first allowing for the effect of the river current. Economists have, however, habitually been guilty of making exactly this error. For example, they have attempted to correlate the cyclical movements of two variables, recording changes in business, without first removing the long time trends due perhaps to currency changes or population growth. They have tried to study the cycles of industry without recognizing the fact that they were dealing with a complex of distinct oscillations of different wave lengths and amplitudes rather than with one simple sine curve. They have asserted that, since the money curve does not correspond with the price curve, prices evidently do not vary exactly and proportionately with the money supply. All of these are clear instances of a failure to recognize the imperative necessity of excluding all forces but the one to be studied, before making any comparisons.

Another illustration may make this necessity more evident: The change in wheat prices in 1916 was due to several independent forces, among which were:

1. Currency inflation.
2. The sinking of wheat in the Atlantic Ocean.
3. The commercial isolation of Russia.
4. Shortage of agricultural labor in Western Europe.
5. Poor climatic conditions for wheat.
6. Increased consumption in soldiers' rations.

A graph of the figures would show that wheat prices did not vary in the same proportion as the money supply. Neither did they vary inversely in the same ratio as the yield per acre. A study of the stocks of wheat on hand each month would show that the price did not conform to the theoretical supply and price curve worked out for this grain. The superficial critics would say that here was first class evidence of the inexactness of so-called economic laws. Yet it was doubtless true that each of the forces governing the price of wheat played its part with the same mathematical exactitude with which the hydrogen and oxygen united to form water. The price of wheat was a resultant of all the forces acting in unison. Why have economists been so slow to recognize the principle here illustrated?

One part of the explanation seems to be that it has been more difficult to separate for measurement the economic than the chemical or physical forces. In the economic field, it has not been feasible to control the conditions of the experiment. Because of this fact, it has been necessary to call upon the science of statistics to differentiate the effects of these separate forces. This new science has responded nobly, but, as yet, it has failed to accomplish the desired end with the precision attained by the chemist or physicist by experimental methods. Fortunately, however, this phase of the science is advancing so rapidly that the outlook for the future appears most encouraging.

If the assumptions just made are correct, if economic and physical phenomena are governed by laws equally exact, there can be no such thing as imperfect or partial correlation. Every correlation to exist must be perfect. Every cause must produce its effect exactly and invariably. If the cause and its effect are once completely isolated, the coefficient must always be unity. In every case, there either is or is not correlation between variable A and variable B—there can be no middle ground.

But it is common to obtain correlation coefficients of 0.3 or 0.6 or 0.7. What do these decimals mean? They merely indicate that the cause and its effect have not been completely isolated—that the effects of conflicting forces still enter in to

mar the results. The low coefficients, then, in no sense indicate imperfections in the actual correlation, but they do show conclusively that the statistician has failed in his efforts to exclude some of the untoward forces which ought to have been eliminated. This failure may have resulted from a non-comprehension of the complexity of the forces involved; from a lack of sufficient information to render the elimination of undesirable forces feasible; or from ignorance of the proper statistical methods which must necessarily be utilized in the process. Those troubled by errors arising from the cause last mentioned would be helped by reading the admirable articles in the June, 1917, issue of THE QUARTERLY PUBLICATIONS OF THE AMERICAN STATISTICAL ASSOCIATION, by Professors Irving Fisher and Warren M. Persons, in which they discuss the merits of some of the various methods of eliminating the effects of the forces which must be excluded before correlating the particular cause and effect under consideration.

The wave motions which are so universal in natural phenomena are just as common in economic fields. These waves have different lengths and amplitudes and the analysis of a compound of several such waves is no simple task. The economist will do well to study carefully the methods used in analyzing light and sound waves and the general characteristics of sine curves before attempting the simplification of the data at hand. He is almost sure to find that his problem is strikingly similar to those with which the physicists have grappled so successfully. Once the undesired forces have been completely eliminated, and the resulting figures have been plotted, two curves emerge—one representing the cause and the other the effect. If the elimination has been entirely successful, the fluctuations in the curves will correspond perfectly and the correlation will be evident at a glance. To compute a coefficient to prove whether correlation does or does not exist is, in this case, a manifest waste of energy.

In attempting by graphic methods, therefore, the correlation of two supposedly related historical variables, two curves are derived which either do or do not resemble each other closely. If they are very similar, all the correlation coefficients in the world will add nothing to the knowledge gained simply

by observing the curves. If they are dissimilar, it may be impossible to say whether there is or is not correlation. In this case, it is easy to tell in advance that a coefficient, if computed, will be low and will, therefore, prove nothing conclusively. In either instance, therefore, the coefficient has added nothing whatever to the knowledge obtainable from the graphic presentation. Too frequently, the coefficients presented are worse than useless for they lend the glamor of apparent erudition to a carelessly made study. Do coefficients, then, have any place in the correlation of historical variables?

So far as the present writer has been able to observe they have but one valid use. The size of the coefficient is merely a means of determining the closeness of fit of two curves. If there is a lag, that is if there is a time interval between cause and effect, it is frequently difficult to determine by the eye the exact lag which has actually occurred. In this case, by trying coefficients with lags of different lengths, it is possible to ascertain the approximate time interval which gives the best fit to the curves—in other words, the most common lapse of time required for the cause to produce the effect. Aside from this purpose, the labor of computing coefficients for historical economic variables seems to be largely wasted. When coefficients are used to cover up careless work in failing to eliminate the causes not under consideration, their use becomes inexcusable.

Strangely enough, many writers still seem to confuse correlation coefficients with the ratio of variation. It is entirely unnecessary for the ratio of variation to approach unity in order to give a high coefficient as the following simple examples will show.

RATIO OF VARIATION UNITY.

Size of Item.	Deviation from Arithmetic Average. x	Deviation Squared. x^2	Size of Item.	Deviation from Arithmetic Average. y	Deviation Squared. y^2	xy
32	−2	4	64	−4	16	+8
33	−1	1	66	−2	4	+2
34	0	0	68	0	0	0
35	+1	1	70	+2	4	+2
36	+2	4	72	+4	16	+8
		10			40	20

$$r_1 = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{20}{\sqrt{10 \times 40}} = \frac{20}{20} = 1.$$

RATIO OF VARIATION 0.5.

Size of Item.	Deviation from Arithmetic Average. x	Deviation Squared. x^2	Size of Item.	Deviation from Arithmetic Average. y	Deviation Squared. y^2	xy
32	−2	4	66	−2	4	+4
33	−1	1	67	−1	1	+1
34	0	0	68	0	0	0
35	+1	1	69	+1	1	+1
36	+2	4	70	+2	4	+4
		10			10	10

$$r_1 = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{10}{\sqrt{10 \times 10}} = \frac{10}{10} = 1.$$

In both instances the coefficients computed by Karl Pearson's method show perfect correlation, although the ratio of variation is entirely different. Further experimentation will demonstrate that changes in the ratio of variation have no effect whatever upon the correlation coefficients.

To sum up, therefore, it seems:

1. That it would be wiser for economists to go more slowly in applying coefficients of correlation to historical economic variables, and that such coefficients should be used mainly as an aid in measuring the length of a lag.

2. That no coefficient equals the graphic method for demonstrating whether correlation does or does not exist.

3. That more work needs to be done in devising statistical methods of eliminating all but one cause before comparing that particular cause with its supposed effect.

4. That the ratio of variation should never be confused with a coefficient of correlation.